This is a free preview of the English edition of *Theory of Errors (Lectures)* by Iosif Refulyevich Entinzon. It contains the title pages, the credits, the complete contents, and Section 1. It stops before Section 2. The page numbers in the contents are the page numbers of the 2023 Russian original.

Iosif Refulyevich Entinzon

Theory of Errors

Lectures

English Edition

English translation presented and published by LCC Research Lab LLC with permission of the author.

Theory of Errors (Lectures)

English edition

Original work: Iosif Refulyevich Entinzon, *Теория ошибок (Лекции)*, Kyiv, 2023. Entinzon remains the author and owner of his work.

English translation presented and published by LCC Research Lab LLC with permission of the author. LCC Research Lab LLC is responsible for this English translation and presentation and does not claim ownership of Entinzon's underlying copyright.

The contents list the headings of the 2023 Russian original. The numbers printed with those headings are the page numbers of that original. They are not the page numbers of this English preview.

This preview contains the title pages, these credits, the complete contents, and Section 1. It stops before Section 2.

I. R. Entinzon

Theory of Errors

(Lectures)

Kyiv — 2023

Contents

The page numbers below are the page numbers printed in the 2023 Russian original. They are not the page numbers of this English edition.

Introduction — 3

The Concept of Probability — 6

Random Variables — 7

Numerical Parameters — 9

The Normal Distribution Law — 11

Problems of Mathematical Statistics — 14

The Concept of an Estimate — 17

Estimates of the Parameters of the Distribution Law: $\tilde{x}$, $\tilde{\sigma}_X^2$, $\tilde{\sigma}_X$ — 20

The Central Limit Theorem — 22

Problem 1. Find the estimate $\tilde{x}$ when the variance σ_X^2 is known — 25

Problem 2. Find the estimate $\tilde{\sigma}_X^2$ when the mathematical expectation $\bar{x}$ is known — 28

Problem 3. Find the estimates $\tilde{x}$ and $\tilde{\sigma}_X^2$ when $\bar{x}$ and σ_X^2 are unknown — 33

The Method of Confidence Intervals — 37

Problem 1. A confidence interval for the estimate $\tilde{x}$ of the mathematical expectation $\bar{x}$ of a normal random variable X when the variance σ_X^2 is known — 39

Problem 2. A confidence interval for the estimate $\tilde{\sigma}_X^2$ of the variance of a normal random variable X when the mathematical expectation $\bar{x}$ is known — 42

Problem 3. Confidence intervals for the estimates $\tilde{x}$ and $\tilde{\sigma}_X^2$ of a normal random variable X when $\bar{x}$ and σ_X^2 are unknown — 48

Procedures — 54

A Numerical Example — 58

The Method of Least Squares — 62

References — 66

Appendix — 67

1. Introduction

Repeated measurements of any quantity always lead to different numerical values. Suppose that a true value of the measured quantity exists. We will then call the difference between the measurement result x and the true value a of the measured quantity the measurement error ε. That is, we set

$$\varepsilon = x - a \tag{1.1}$$

Depending on the causes that produce them, measurement errors are divided into random errors, systematic errors, and gross errors (blunders). Random errors are those whose values change from one measurement to another. The magnitude of these errors cannot be established before the experiment. They arise from inaccuracy of measurement — random errors of the measuring instruments, random errors of the experimenter, and inexact adherence to the measurement procedure — and from the measured quantity itself not being constant.

Systematic errors differ from random errors in that their value remains constant through a series of measurements of the same kind. They can therefore be excluded from the final result if their magnitude has been determined by a prior study of the measurement procedure.

The division of errors into systematic and random is plainly conditional, because the same errors should be treated as random in some cases and as systematic in others. For example, a series of measurements made at the same temperature will contain a systematic error which arose because the temperature correction is not taken into account (or is taken into account incorrectly). However, in a series of measurements made at different (not fixed) temperatures, those same errors will be random in character. Likewise, an unaccounted error of a standard

(errors of calibration) will be systematic in character for a given laboratory; however, when results obtained in different laboratories are processed, that error must be regarded as random.

Gross errors (blunders) are also random, but their character differs substantially from that of random measurement errors. This difference is due to the fact that if random measurement errors arise when the apparatus is working properly and the experimenter is acting correctly, then gross errors are caused by faulty measuring equipment or by errors in the experimenter's work. If the gross errors are considerable, the results stand out sharply from the others and should be discarded. If that is not the case, the results may also be discarded, but only after special quantitative criteria have been applied.

If the quantity measured is the one that directly interests the experimenter, the measurements are called direct. If the quantity to be determined is calculated, through a given functional relation, from quantities obtained by direct measurement, the measurements are called indirect. When several measured quantities are connected by a known functional relation — for example, the three angles of a triangle; the temperature, volume, and pressure of a gas; and so on — the values obtained will, because of measurement errors, generally contradict one another. Such measurements are called conditional.

In direct, indirect, and conditional measurements, the errors may be systematic, random, or gross.

Any real measuring instrument, working in real situations, is subject, as is the situation itself (the setup), to the uncontrolled influence of many external and internal factors. The measured quantity can therefore take different numerical values.

Statisticians joke about this, saying that the measured quantity α is a function of an infinite number of parameters Φi:

$$\alpha = \alpha(\Phi 1, \Phi 2, \Phi 3, \Phi 4, \ldots, \Phi 40, \ldots, \Phi 400, \ldots, \Phi\infty). \tag{1.2}$$

In theory, an infinite (∞) number of parameters have an effect; in practice, 400. Of these, the experimenter can name 40, if he thinks the situation through from all sides. He is able to control — to hold fixed — only 4 parameters, and even those only to within the accuracy of the measuring instrument.

That joke contains the chief feature of any measurement. The number of factors that affect the measured quantity is enormous, and their influence is uncontrolled and random. The measured quantity should therefore be regarded as a random variable. The true quantity, as is evident, cannot be obtained from the experiment, and so statistics has no concept of a “true quantity.”

Yet, “there is no harm without some good” — the fact that the number of influencing parameters is infinite (or enormous) turned out to be decisive in formulating objective principles for discussing the results of direct measurements. These lectures are devoted to considering these principles.

We will call a random variable a quantity that, as the result of an experiment, can take different numerical values. A random variable is the possibility that a number appears on the screen. A particular number on the screen is a realization of the random variable. A random variable may take discrete values, or any values in a given interval: it may be a discrete or a continuous random variable. From here on, only continuous random variables will be considered.

In the theory of errors, random variables are the principal object of study. The principal mathematical apparatus of the theory is therefore probability theory, and the mathematical statistics that follows from it.

The preview ends here, before Section 2.